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ABSTRACT

This study explores the potential effectiveness of a new procedure for identifying and studying certain of the cognitive processes used during problem solving. The procedure is used in an attempt to categorize the types of conceptual thinking problem solvers employ, to study trial-and-error behavior, and to investigate problem solvers' abilities to coordinate multiple bits of information. Plastic blocks are used in the tasks; one fifth-grade and one seventh-grade class were the subjects. Results indicated that five distinct types of strategies were used by the students; specific information on aspects of their work is also given. (MS)

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A Procedure for Studying the Cognitive Processes Used During Problem Solving: An Exploratory Study

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A Procedure for Studying the Cognitive Processes Used During Problem Solving: An Exploratory Study*

Statement of the Problem

The cognitive processes used by children during problem solving have received considerable attention in mathematics education research in recent years. The identification of the processes used by both successful and unsuccessful problem solvers has been hampered by the difficulties involved in making accurate observations of mental processes which may or may not have physical manifestations during problem solving. In addition problem solvers themselves often are unaware of many of the thought processes they use while attempting to solve problems. The most popularly used methods for studying problem solving processes are not sufficiently reliable or valid. There is a need to devise procedures which enable accurate recording of problem solving processes. This study explores the potential effectiveness of a new procedure for identifying and studying certain of the cognitive processes used during problem solving. In addition, the procedure is used to attempt to categorize the types of conceptual thinking problem solvers employ, to study the use of trial-and-error behavior, and to investigate problem solvers' abilities to coordinate multiple bits of information.

Rationale

Mathematical problem solving has been the object of more attention by mathematics education researchers in recent years than any other topic in the mathematics curriculum. With this growing interest in problem solving has come an awareness of the importance of identifying patterns of behavior in addition to noting simply that a problem was or was not solved, how much time was taken, how many errors were made, and so on. While a number of different approaches have been employed to study cognitive processes, the most common have been of two types:

1. Have problem solvers "think aloud" as they work on problems. The investigator

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records their behaviors by means of some sort of coding scheme developed for the purpose of identifying the use of certain key processes. With the aid of video and/or audio tape recordings, the protocols of a subject are studied across many problems as well as protocols of several subjects for a particular problem.

2. Problem solvers are asked to reflect on what they did and why after a problem solving episode has been completed. Such retrospective introspection is used generally to validate judgments made by the investigator which were based on observable behavior during problem solving.

Unfortunately protocol analysis based on either of these two approaches has serious weaknesses. The first approach can be attacked on the grounds that forcing the problem solver to think aloud during problem solving may have a deleterious effect on performance. At the same time there is a tendency for problem solvers to talk only about moves which they believe to be safe or correct. Retrospective analysis is often criticized because it is unreliable inasmuch as it is unlikely that accurate accounts of behaviors, including all of the cognitive processes employed, can be reconstructed. Finally, each of these approaches may in fact be a test of a person's linguistic development rather than a study of the presence or absence of a cognitive process.

An ideal situation would be to devise problem solving tasks which force the subject to manifest mental processes through the physical manipulation of objects thereby enabling an observer to record the processes being used. Thus, the need for the subjects to verbally relate their thoughts or accurately remember what was done would be removed. Efforts in this direction led to a consideration of the techniques used by psychologists to study concept learning. The famous sorting task developed by Vygotsky (1962) which was intended as a means to explore the correspondence between the way people organize their thinking and language embodies the essential characteristics of a good procedure for observing problem solving processes. The procedure devised for this study is a modification of Vygotsky's task. The value of the task lies in the possibility of observing directly certain of the problem solver's cognitive

processes. The nature of the task allows problem solvers to be comparatively free in their procedures for attacking the problem, but at the same time the physical moves made reflect several aspects of their reasoning: forming hypotheses, testing and discarding them, attention to available information and so on. The whole course of the task: comprehension of the problem, testing conjectures and responding to additional information, and ultimately finding a solution--provides a number of indicators of problem solving ability or the lack of it.

Conjectures

Three conjectures were explored:

1) The procedure devised is a valid and reliable method for studying certain cognitive processes used during problem solving.

2) The use of trial-and-error behavior decreases with increasing age.

3) The ability to coordinate multiple conditions (bits of information) increases with increasing age.

Conjectures 2) and 3) were selected because there is substantial evidence to support them. It was decided that since conjecture 1) was to receive primary focus, one way to validate the procedure would be to test the extent to which conjectures 2) and 3) were observed using that procedure.

Because of the exploratory nature of the study no additional conjectures were made. Instead the decision was made to attempt to categorize the types of strategies used and to look for patterns of problem solving behavior.

Methods

Materials used in procedure consisted of plastic blocks selected from a sixty piece set of attribute logic blocks made by Invicta Educational Aids. The blocks varied in color, shape, size, and thickness. There were two colors (blue and yellow), four shapes (square, circle, triangle, and hexagon), two sizes (large and small), and two thicknesses (thick and thin). For purposes of making the task more complex,

eight of the original thirty-two blocks were removed (two of each shape) leaving a set of twenty-four blocks (A complete description of the materials is available upon request).

On one side of each block was written one of four letters: A, B, C, or D. A was written on all large squares and circles; B on all small squares and circles; C on all large triangles and hexagons; and D on all small triangles and hexagons. Students who participated in the study were told they were going to play a game with the blocks. At the beginning of the game the blocks were scattered randomly on a table top letter-side down. A student was told that there were four different letters written on the "bottom" of each block and that all the blocks with the same letter on them "belong together for some reason." The task was to attempt to put the blocks into four piles so that all the blocks in a given pile had the same letter. The students were instructed that only the investigator was allowed to turn over any blocks. The object of the game was for them to put all the blocks into their correct piles before the investigator turned over all the blocks. Four cards, each having one of the four letters on it, were placed on the table to assist the student in sorting the blocks.

To begin, one block was turned over (an A block) and the student was instructed to use this single piece of information to sort the blocks into four piles. After the initial sorting was completed, the investigator selected one incorrectly placed block, turned it over, and placed it where it belonged. The student then performed another sort (called a move or step). After each move the investigator turned over an incorrectly placed block. This procedure continued until the student had placed all blocks into their correct pile. When the task was completed the student was asked to explain the rule for classifying the blocks. Finally, the student was asked to make up a different

rule for classification and the roles were reversed - the student gave information to the investigator who then attempted to classify the blocks and discover the student's rule.

Throughout the game each move made by a student was recorded by the investigator using a coding sheet developed for this study. In addition audio or video tape recordings were made in order to check the accuracy of the coding.

Originally students in grades 4,6,8, and 10 were to participate in the study. Unfortunately, due to unavoidable difficulties it was impossible to use these grades. Instead, because of their availability a class of grade seven middle school students and a class of grade five students in a feeder elementary school were used. Students were of average to well-above average ability. The schools were in an area of Bloomington, Indiana which has a pre-dominance of university faculty and student families.

Preliminary Analysis and Results

Due to the exploratory nature of this study and the clearly disparate performance of the two groups with respect to various quantitative measures it was both inappropriate and unnecessary to perform any statistical analyses to determine the level of significnace of differences between the groups.

Quantitative Analysis

As expected seventh graders outperformed fifth graders in every aspect of the task. This fact is made evident by a comparison of the two groups with respect to the number of "moves to solution" and the "time to solution." Table 1 presents this comparison.

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insert Table 1 here

Analysis of the students' protocols indicated that five distinct types of strategies were used. The five strategies and a description of each follows:

1) Trial-and-error (TE) - Random or apparently random arrangement of blacks into groups; no discernible rule employed.
2) Heterogeneous groupings (HG) - Attempt to place one block of each shape (or another attribute) in each group; student tends to focus attention within a single group not between groups. [this strategy is similar to Vygotsky's designation, "collections" (Vygotsky, 1962)]
3) Local classification (LC) - Student uses given information for a single group to place blocks in that group; student does not coordinate information between 2 or more groups; student focuses on similarities between individual blocks, not among all blocks.
4) Partial, global classification (PC) - student coordinates information at first but is unable to complete the classification for all four groups; that is, the classification breaks down after sorting blocks into two or three

groups or after sorting some, but not all, of the blocks into all four groups.

5) Global classification (GC) - Highest level of performance; student coordinates all information and performs a logically consistent classification of blocks into all four groups.

Table 2 displays the percent of each type of strategy used at each grade level. The most noteworthy feature of these data is the marked increase in the use of the global classification strategy and the dramatic decrease in reliance on trial-and-error and local classification strategies from grade five to grade seven.

Insert Table 2 here

The literature on mathematical problem solving devotes considerable attention to variables related to understanding the problem and ability to make generalizations after finding a solution to a problem. Certain features of these two aspects of problem solving were investigated for the task of this study. Understanding the problem was measured by considering the types of questions, if any, students asked before the game began and by the type of strategy used for the initial move. Ability to generalize the solution was judged by the students' abilities: (1) to verbalize the principle for classification after the task was completed; and (2) to devise a different and non-trivial principle of classification for the task. Summary data for these variables are given in Tables 3 and 4.

Insert Table 3 here

Insert Table 4 here

The data in Table 3 indicate that with only one exception all students had at least a partial understanding of what the task involved. The two students in grade seven who had a somewhat hazy understanding of the object of the task stated that their misunderstanding at the beginning of the game was due to lack of concentration

when the game rules were discussed. Among the six fifth graders who demonstrated only a partial understanding of the task two indicated they did not "catch on at first." These two students ranked among the upper 50% of all grade five students in overall performance on the task. The other four students in this category appeared never to gain a full understanding of the task. In fact, two of these four seemed quite surprised when they arrived at a correct solution. The student who had no apparent understanding of the task also used trial-and-error throughout the course of the game and was unable to articulate the general principle for classification upon completing the task even with prompting by the interviewer.

On the basis of the two criteria for ability to generalize listed above, students were grouped into three categories: conceptual, transitional, and pre-conceptual. Conceptual students satisfied both criteria; transitional students exactly one criterion; and pre-conceptual students satisfied neither of the two criteria. As expected most (86%) of the seventh graders were at the conceptual level, whereas only 35% of the fifth graders were conceptual and another 39% were transitional.

A final quantitative measure was the efficiency of the strategies used. According to Bruner "...the best approach to understanding the growth of cognitive processes would be in a study of the nature of efficiency and inefficiency in the strategies employed by human beings in thinking and in solving problems" (Bruner, Olver, and Greenfield, 1966, pp. viii, ix). To this end an efficiency score was calculated for each student. The efficiency was defined as the ratio of non-redundant global classification moves to total moves. Thus, if E_i represents the mean efficiency score for grade i, $0 \leq E_i \leq 1$. Also, $E_5 = .16$ and $E_7 = .50$. These scores reinforce the fact that older students were more efficient problem solvers insofar as they used a greater percentage of global classification moves.

Non-Quantitative Analysis

The foregoing quantitative data provide a somewhat superficial and obvious indication of the nature of the differences between fifth and seventh grade students on the experimental task. These data suggest, for example, that the younger students

use a trial-and-error strategy more often than the older students but they give no indication as to the conditions under which trial-and-error was used. Consequently, the students' protocols were analyzed in an attempt to identify any discernible patterns of behavior with respect to: (1) the conditions under which a trial-and-error strategy was used; (2) the extent to which students were able to coordinate multiple conditions (i.e. two or more bits of information).

While a stringent and thorough protocol analysis has not been completed at the time of this writing, certain observations appear to be generally true. These observations include the following:

(1) Students who used trial-and-error as their dominant strategy were unable to articulate the general principle for classification after the problem was solved (Grade 5 only).

(2) Students who used trial-and-error only occasionally (less than 10% of the time) used it only after three or more bits of information had been provided and they were able to state the general principle for classification (Grade 5 only).

(3) Students who found it difficult to coordinate multiple conditions tended to focus on similarities between individual blocks (i.e., use a local classification strategy) before they resort to trial-and-error (Grades 5 & 7).

(4) Grade seven students used trial-and-error very infrequently. Even when they were stumped they tended to use either local or partial classification instead of trial-and-error.

(5) When grade seven students used trial-and-error, it was often used "strategically". That is, it was used as a tactic to get more information quickly rather than merely as a result of confusion or misunderstanding.

(6) There was a tendency on the part of several grade seven students to look for a more complex principle of classification than the one used. For example, some of the more capable students stated that they expected the "rule" to be harder than it was. A common comment from these students was

"It was too easy! I didn't think you would have such an easy rule." Two students (one fifth and one seventh grader) observed that the problem could be made harder by having more shapes than groups and different numbers of blocks in each group.

Discussion

In general the preliminary results of this study offer strong support for the findings of Vygotsky (1962) and several others which suggest that prior to adolescence children are incapable of purely conceptual thinking on a consistent basis. Since mathematical problem solving of the type which requires simultaneous coordination of multiple conditions is an integral part of the intermediate and middle grades curriculum, great care should be taken to match problems to the developmental characteristics of children. In addition, teachers and curriculum developers should become aware that the objectives of problem solving instruction must vary depending on the students' levels of conceptual development. That is, whereas seventh graders may be expected to develop considerable success in solving certain multiple condition problems, students in the fifth grade may benefit from exposure to such problems but should not be expected to demonstrate much success unless specific instruction is provided.

As is usually true of exploratory research many questions are raised but few, if any, answers are obtained. This study is no exception. The following questions for future investigation have arisen from the preliminary analysis.

1) One strategy that has emerged from the mathematical problem solving literature as a widely used strategy is that of trial-and-error. While this strategy was used frequently by students in this study it was not as common as expected. Is it possible that what appears to be trial-and-error behavior is actually a primitive form of a more sophisticated strategy?
2) What level of cognitive strain is necessary before a student at a particular level will resort to trial-and-error behavior?

3) Fifth grade students are unable generally to coordinate 3 or more conditions (bits of information). This claim is consistent with earlier obervations made by the author (Lester, 1978). At what developmental level do students acquire the ability to coordinate multiple conditions? Is this ability acquired prior to grade 7?

4) According to Bruner (Bruner et al, 1956), cognitive strain is a function of memory and inference. To what extent is inability to coordinate multiple conditions a function of one or both of these factors?

5) Students who frequently used the local classification strategy appeared also to be low risk takers. To what extent is the strategy a student uses a function of variables such as willingness to take risks, tolerance of ambiguity, and stress?

Future Study

Although the quantitiative analyses yielded no surprising results, the non-quantitative protocol analysis appeared to offer valuable insights into the nature of the strategies students employ. Of course the task used in this study is representative of only one of many types of mathematics problems; consequently care must be taken not to generalize behavior beyond this task. Nevertheless, the prospect is promising for gaining valuable information from protocol analysis of problem solving of this type. In addition to the analysis procedures used herein, other potentially fruitful procedures might include state-space analysis (Goldin & Luger, 1977) or "search-space" analysis (Harik, in press) to investigate the behavior paths of problem solvers.

Finally, the interaction between problem and problem solver should be studied. That is, a particular problem should be used with children of a wider range of developmental levels (e.g., grades 3,5,7,9), and different but similar problems should be administered to the same students.

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Table 1

Mean Number of Moves to Solution and Mean Time to Solution

Grade	*n*	Moves to Solution	Time to Solution[a]
5	23		
M		11.6	25.4
SD		4.9	10.8
7	21		
M		6.5	21.1
SD		2.1	10.5

[a]Time was recorded to the nearest minute.

Table 2

Percent Useage of Each Strategy Type

Grade	TE[a]	HG[b]	LC[c]	PC[d]	GC[e]
5	27.3	3.0	34.7	23.6	11.4
7	8.1	6.4	7.2	33.9	44.3

Note: Row total for Grade 7 less than 100% due to rounding error.

[a]TE - trial-and-error

[b]HG - heterogeneous grouping

[c]LC - local classification

[d]PC - partial, global classification

[e]GC - global classification

Table 3

Level of Understanding of Task (%)

Grade	*n*	Full	Partial	None Apparent
5	23	69.6	26.1	4.3
7	21	90.5	9.5	0

Table 4

Level of Ability to Generalize Principle of Classification (%)

Grade	*n*	Conceptual	Transitional	Pre-Conceptual
5	23	34.8	39.1	26.1
7	21	85.7	14.3	0